TILE PATTERNS & GRAPHING

THE BIG IDEA

Tile patterns provide a meaningful context in which to generate equivalent algebraic expressions and develop understanding of the concept of a variable. Such patterns are a useful context for developing techniques for solving equations and for introducing the concept of graphing.

START-UP

Overview

A tile pattern provides the context for generating equivalent expressions, formulating equations, and creating bar graphs.

Materials

- Red and black counting pieces, 60 per student.
- Start-Up Master 1.1, 2 copies per student and 1 transparency.
- Start-Up Master 1.2, 1 transparency (optional).
- Start-Up Master 1.3, 1 copy per student and 1 transparency.
- Start-Up Master 1.4, 1 copy per student and 1 transparency.
- 1/4″ grid paper (see *Blackline Masters*), 1 sheet per student and 1 transparency.
- Black counting pieces for the overhead.

FOCUS

Overview

Tile patterns are used to generate equivalent expressions, formulate equations, solve equations, and introduce coordinate graphs.

Materials

- Red and black counting pieces, 25 per student.
- 1/4″ grid paper (see Appendix), 2 sheets per student and 1 transparency.
- Focus Master 1.1-1.2, 1 copy of each per student and 1 transparency.
- Focus Master 1.3, 1 copy per student.
- Black counting pieces for the overhead.

FOLLOW-UP

Overview

Students find patterns in arrangements of tile, write equivalent algebraic expressions for the number of tile in the *n*th arrangement, and create coordinate graphs to show the number of tile in certain arrangements.

Materials

- Follow-Up 1, 1 copy per student.
- Square tile or counting pieces (optional) for student use at home.
- 1/4″ grid paper (see Appendix).

TEACHER NOTES

START-UP

Overview

A tile pattern provides the context for generating equivalent expressions, formulating equations, and creating bar graphs.

Materials

- Red and black counting pieces, 60 per student.
- Start-Up Master 1.1, 2 copies per student and 1 transparency.
- Start-Up Master 1.2, 1 transparency (optional).
- Start-Up Master 1.3-1.4, 1 copy of each per student and 1 transparency.
- 1/4″ grid paper (see Appendix), 1 sheet per student and 1 transparency.
- Black counting pieces for the overhead.

ACTIONS

1 Distribute counting pieces to each student or group of students. Display the following sequence of 3 tile arrangements on the overhead. Have the students form this sequence of arrangements. Then have them form the next arrangement in the sequence. Ask the students to leave their sequence of arrangements intact so it can be referred to later.

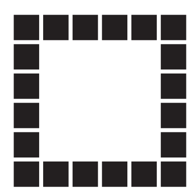
2 Ask the students to consider the sequence of arrangements in which the 4th arrangement is the one illustrated in Comment 1. Ask them to imagine the 20th arrangement and to determine the number of tile required to build it. Ask for a volunteer to describe their method of determining this number. Illustrate their method on the overhead, using a transparency of Start-Up Master 1.1.

COMMENTS

1 In this and subsequent activities, the counting pieces are often referred to as *tile*. In later activities the color of the counting pieces is relevant; however, color is not relevant in this activity.

Many students will form the 4th arrangement as shown to the right. If someone forms another arrangement, acknowledge it without judgment, indicating there are a number of ways in which a sequence can be extended.

In Action 7 the students will be asked to convert their sequence of arrangements into a bar graph.



2 There are various ways to arrive at the conclusion that 84 tile are required to build the 20th arrangement. Here is one possible explanation: "There are just as many tile between the corners of an arrangement as the number of the arrangement, for example, in the 3rd arrangement there are 3 tile between the corners, in the 4th arrangement there are 4 tile between corners, and so forth. So in the 20th arrangement there will be 20 tile between corners on each side. Since there are 4 sides and 4 corners, there will be 4 times 20 plus 4 tile." This way of viewing tile arrangements can be illustrated as shown below.

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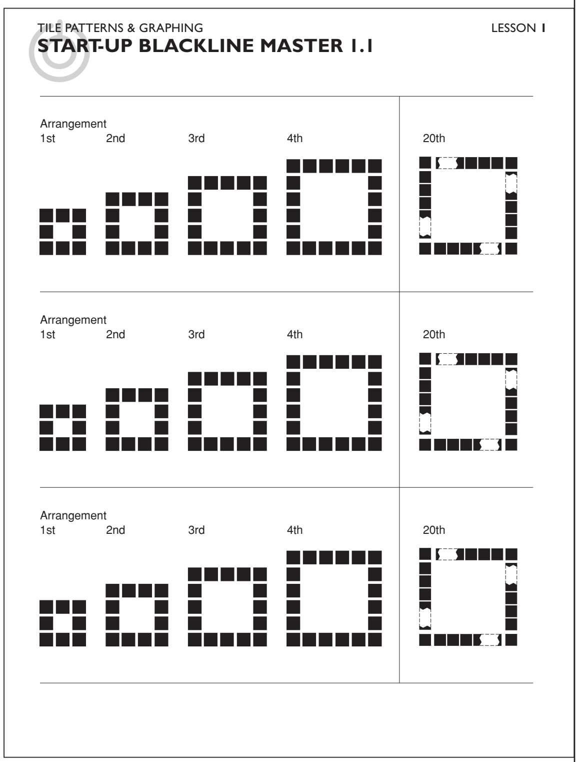
START-UP

ACTIONS


TILE PATTERNS & GRAPHING
LESSON 1
START-UP BLACKLINE MASTER 1.1

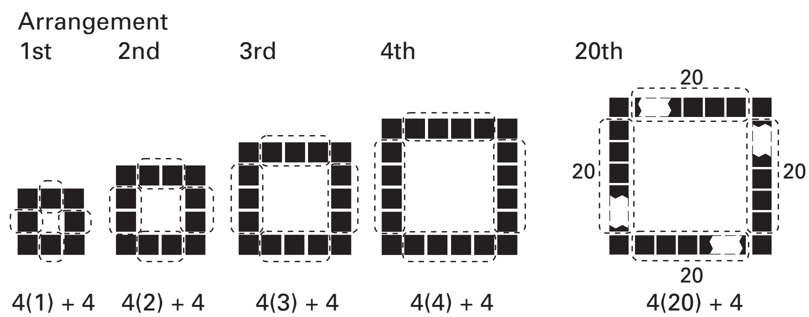
Arrangement
1st 2nd 3rd 4th 20th

Arrangement
1st 2nd 3rd 4th 20th

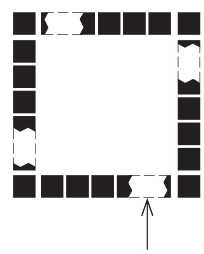
Arrangement
1st 2nd 3rd 4th 20th

COMMENTS

2 continued



Some students may write formulas to represent the number of tile in any arrangement. If so, you might ask them to relate their formula to the 20th arrangement. This will be helpful for other students who need time to work with specific cases before generalizing in Action 4.



Note that the sections of white space in the strips of tile forming the 20th arrangement on Master 1.1 are intended to suggest there are missing tile in the arrangement. That is, by mentally elongating the strip, one can imagine it contains 20 tile.

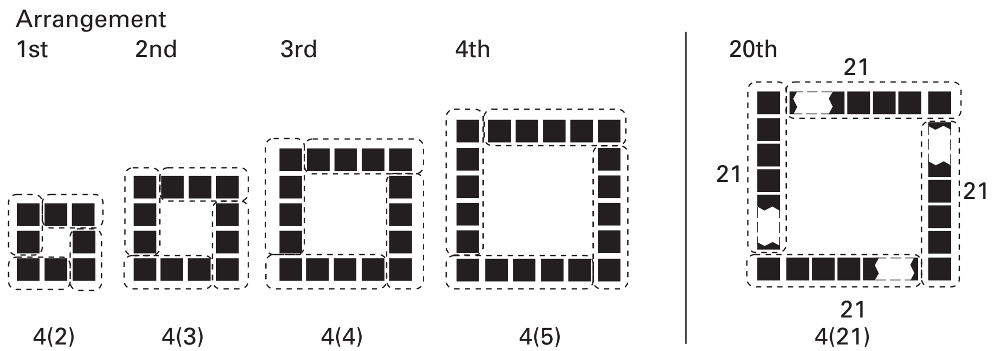
3 Distribute 2 copies of Start-Up Master 1.1 to each student. Have the students record on a copy of Start-Up Master 1.1 the method of viewing the arrangements described in Comment 1. Then ask for volunteers to describe other ways of determining the number of tile in the 20th arrangement. Illustrate these methods on the overhead, using a transparency of Start-Up Master 1.1. Have the students make a record of these methods on their copies of Start-Up Master 1.1. Continue until 5 or 6 different methods have been recorded.

3 Normally, in a group of 25 to 30 students, there will be several different methods proposed for determining the number of tile in the 20th arrangement. In order to obtain a variety of ways of viewing the arrangements beyond those suggested by students, you can ask the students to devise additional ways, or you can devise other ways. Shown on the two following pages are 5 ways of counting the number of tile in the 20th arrangement.

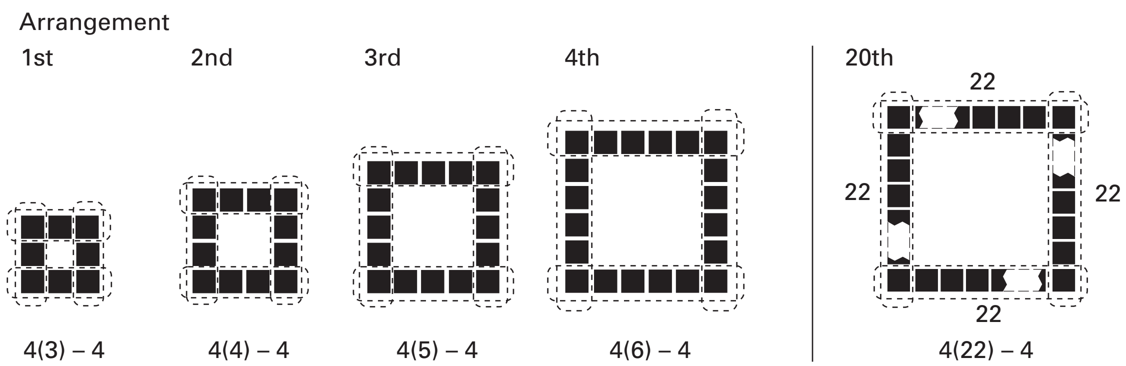
START-UP

ACTIONS

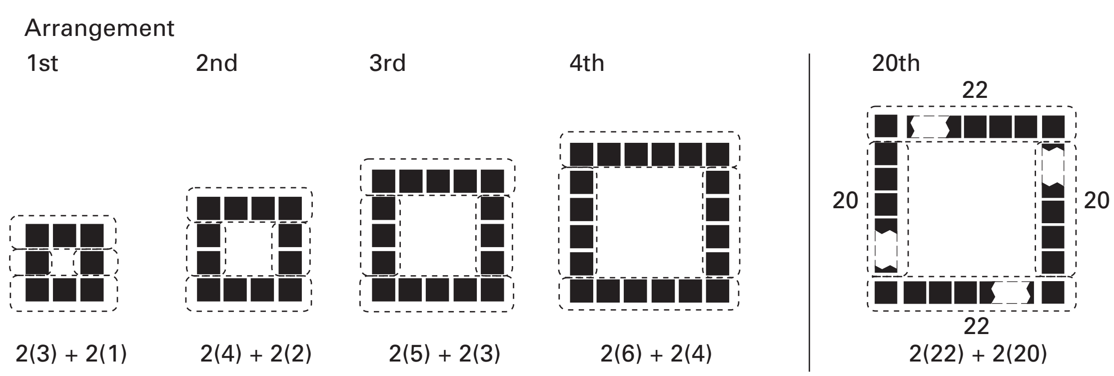
COMMENTS



There are 21 tile on each side, starting at one corner and ending before the next corner.



There are 22 tile on each side, counting each corner twice.



There are 22 tile on the top and the bottom and 20 on each side between the top and the bottom.

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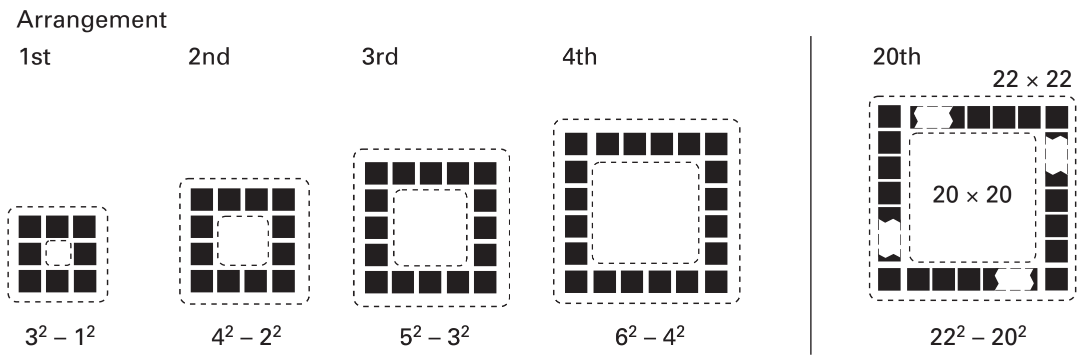
START-UP

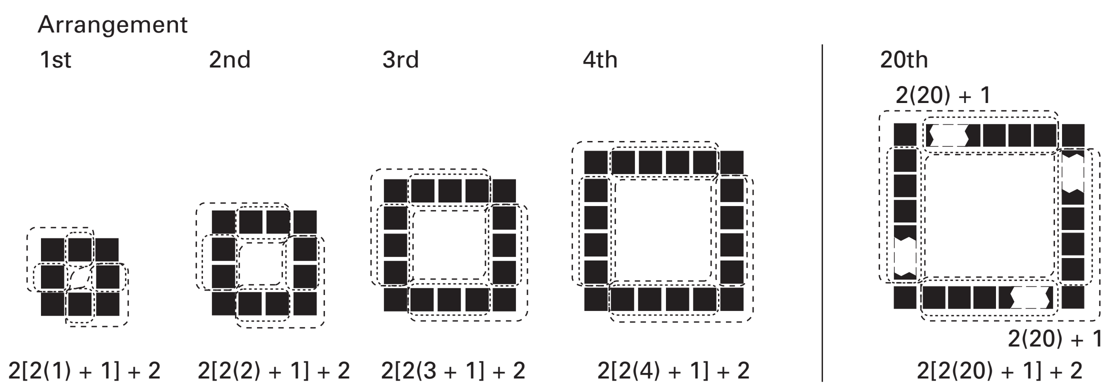
ACTIONS

COMMENTS

3 continued



There is a 22 x 22 square with a 20 x 20 square removed from inside it.



There are 2 "L-shapes," each containing 2 times 20 plus 1 tile, and 2 corners.

4 Tell the students that one of the things discovered about the arrangements is that the number of tile in the bottom row of an arrangement contains two more tile than the number of the arrangement and, thus, the following statements are true:

Arrangement 1 contains 1 + 2 tile in the bottom row.

Arrangement 2 contains 2 + 2 tile in the bottom row.

Arrangement 3 contains 3 + 2 tile in the bottom row.

4 The statements can be written one at a time on the overhead as you make them or you can prepare an overhead transparency from Start-Up Master 1.2 and reveal the statements one at a time as you make them.

A *variable* is a letter used to designate an unspecified or unknown number. Variables allow for great economy in mathematical discourse. In this instance, the use of a variable enables one to replace an infinite collection of statements with a single statement.

Initially, some students may not easily grasp the concept of variable. However, as variables become a part of classroom discussion, these students generally come to understand and use them appropriately.

START-UP

ACTIONS

Arrangement 4 contains 4 + 2 tile in the bottom row.

Arrangement 5 contains 5 + 2 tile in the bottom row.

Arrangement 6 contains 6 + 2 tile in the bottom row.

Comment that one could continue making such statements indefinitely.

Point out that the statements all have the same form, namely:

Arrangement __ contains __ + 2 tile in the bottom row, where the blank is filled by one of the counting numbers, 1, 2, 3, 4, ….

Tell the students that in mathematical discourse, instead of using a blank, it is customary to use a letter, for example:

Arrangement n contains n+ 2 tile in the bottom row, where n can be replaced by any one of the counting numbers 1, 2, 3, 4, ….

Introduce the term *variable*.

5 Distribute a copy of Start-Up Master 1.3 to each student (see following page). For one of the methods of viewing arrangements discussed above, illustrate how the nth arrangement would be viewed for that method. Then write an expression for the number of tile in the nth arrangement. Ask the students to do this for the other methods discussed. For each method, ask for a volunteer to show their sketch and corresponding formula. Discuss.

COMMENTS

5 Shown on the following page are illustrations for the 6 methods described earlier, with corresponding formulas. During the discussion, you can point out notation conventions which may be unfamiliar to the students, such as the use of juxtaposition to indicate multiplication, e.g., $4n$, and the use of grouping symbols such as parentheses to avoid ambiguities, e.g., writing $4(n + 1)$ to indicate that $n + 1$ is to be multiplied by 4 in contrast to writing $4n + 1$ which indicates that n is to be multiplied by 4 and then 1 is to be added to that product.

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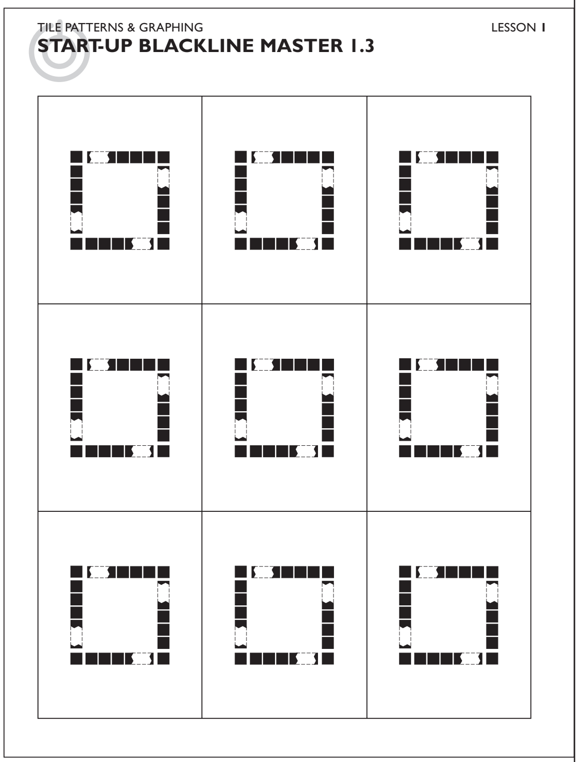
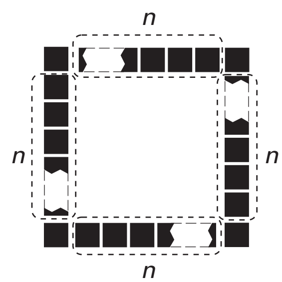
START-UP

ACTIONS

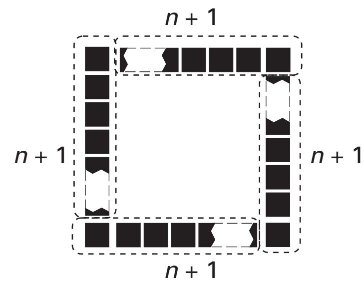


COMMENTS

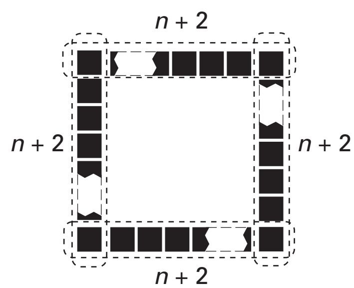
5 continued



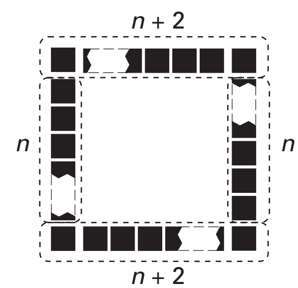
$4n + 4$



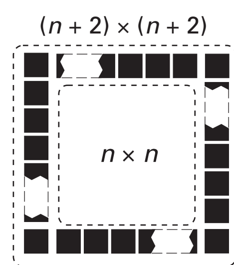
$4(n + 1)$



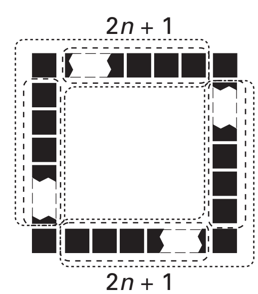
$4(n + 2) - 4$



$2(n + 2) + 2n$



$(n + 2)^2 - n^2$



$2(2n + 1) + 2$

If desired, the transparency can be cut apart so that, on the overhead, an *n*th arrangement can be placed alongside its corresponding 20th arrangement.

START-UP

ACTIONS

6 List all the expressions obtained for the number of tile in the *n*th arrangement. Discuss *equivalent expressions*.

7 Tell the students that one of the arrangements requires 200 tile to build. Ask them to determine which arrangement this is. Discuss the methods the students use. Relate the students' work to solving equations. Repeat, as appropriate, for other numbers of tile.

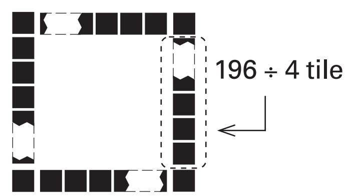
COMMENTS

6 Here are expressions for the number of tiles in the *n*th arrangement, as illustrated above: $4n + 4$, $4(n + 1)$, $4(n + 2) - 4$, $2(n + 2) + 2n$, $(n + 2)^2 - n^2$, $2(2n + 1) + 2$.

Expressions, such as those listed, which give the same result when evaluated for any possible value of n, are said to be *equivalent*. They are also said to be *identically equal* or, simply, *equal*.

Which form of equivalent expressions is preferable depends upon the situation. For example, when n is 99, the second of the above expressions is easy to evaluate, while when n is 98, the third may be preferable.

7 Various methods can be used. Viewing the arrangement as described in Comment 2 suggests removing the 4 corner tile and dividing the remaining 196 by 4. Thus, it is arrangement number 49 that contains 200 tile. A sketch such as the one shown below may be helpful.



The above line of thought can be given an algebraic cast. The number of tile in the *n*th arrangement is $4 + 4n$. Thus, one wants the value of n for which $4 + 4n = 200$. Excluding the 4 corner tiles reduces $4 + 4n$ to $4n$ and 200 to 196. Thus, $4n = 196$ and, hence, $n = 49$.

A statement of equality involving a quantity n, such as $4 + 4n = 200$ is called an *equation in n*. Determining the quantity n is called *solving the equation*.

Other ways of viewing the arrangement may lead to other methods of determining its number. For example, viewing the arrangement as described in the first method of Comment 3 may lead to dividing 200 by 4 and noting that the result, 50, is one more than the number of the arrangement. This, in effect, is solving the equation $4(n + 1) = 200$.

continued next page

START-UP

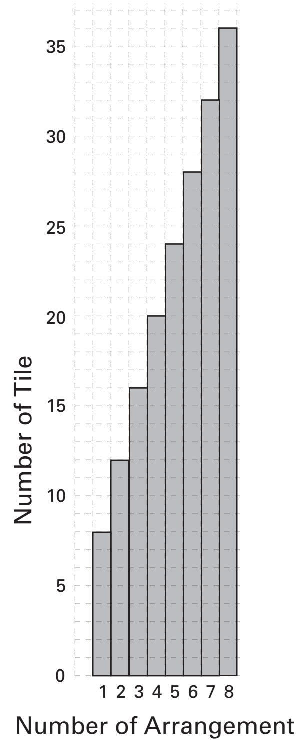
ACTIONS

8 Ask the students to convert the sequence of 4 arrangements in Action 1 to portray a bar graph showing the numbers of tile in these 4 arrangements, as illustrated below. Then distribute one sheet of 1/4″ grid paper to each student and ask the students to draw a bar graph to illustrate the number of tile in the first 8 arrangements of this sequence. Discuss the students' observations about the graph.

COMMENTS

8 By rearranging the tile to portray a bar graph, students are more likely to see the relationship between the sequence and their bar graph.

Shown below is a bar graph showing the number of tile in the first 8 arrangements.



Observations about the graph may be varied. Here are a few examples:

Each bar is 4 squares higher than the previous bar.

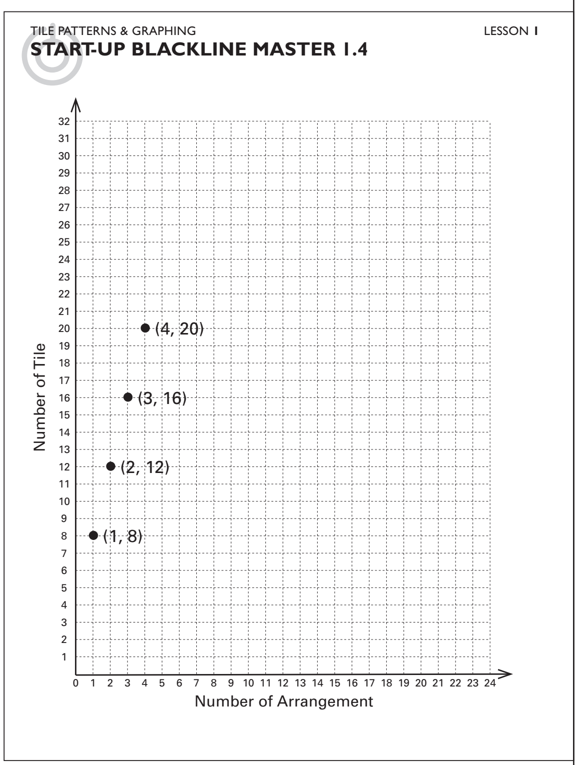
The increase from bar to bar is always the same.

The number of squares in each bar is a multiple of 4, starting with 8.

START-UP

ACTIONS

9 Give each student a copy of Start-Up Master 1.4 and tell them this is a *coordinate graph* of the first 4 arrangements of the sequence. Ask the students to discuss their ideas about how the graph was formed and where on the graph they think points for other arrangements in the sequence would lie. Have volunteers show their ideas on a transparency of Master 1.4.



COMMENTS

9 The coordinates of a point on the graph are an ordered pair of numbers, the first of which tells how many units to count from zero along the horizontal axis (in this case, how many units to count to the right of zero, identifying the arrangement number). The second coordinate in an ordered pair tells how many units to count from zero along the vertical axis (in this case, above zero, identifying the number of tile in the arrangement). It is customary to label the horizontal and vertical axes by the quantities they represent. (Note that grid lines, not spaces, are numbered.)

Some possible observations students may make about the graph include:

The points of the graph lie on a straight line.

The points are equally spaced.

To get from one point to the next, go 1 square to the right and 4 up.

The increase from point to point is always the same.

There are only points on the graph where n is an integer.

Some students may draw a line connecting the points of the graph. Note that, while doing so is okay, it does imply there are arrangements for non-integral values of n. That is, it suggests there are arrangements numbered $2\frac{1}{2}$ or $4\frac{1}{3}$, for example. The students may even suggest ways of constructing such arrangements. However, note that throughout this lesson, each graph is a set of discrete points since n is always viewed as a counting number.

TEACHER NOTES

START-UP BLACKLINE MASTER 1.1

Arrangement
1st 2nd 3rd 4th 20th

Arrangement
1st 2nd 3rd 4th 20th

Arrangement
1st 2nd 3rd 4th 20th

START-UP BLACKLINE MASTER 1.2

Arrangement 1 contains 1 + 2 tile in the bottom row.

Arrangement 2 contains 2 + 2 tile in the bottom row.

Arrangement 3 contains 3 + 2 tile in the bottom row.

Arrangement 4 contains 4 + 2 tile in the bottom row.

Arrangement 5 contains 5 + 2 tile in the bottom row.

Arrangement 6 contains 6 + 2 tile in the bottom row.

Arrangement 7 contains 7 + 2 tile in the bottom row.

Arrangement 8 contains 8 + 2 tile in the bottom row.

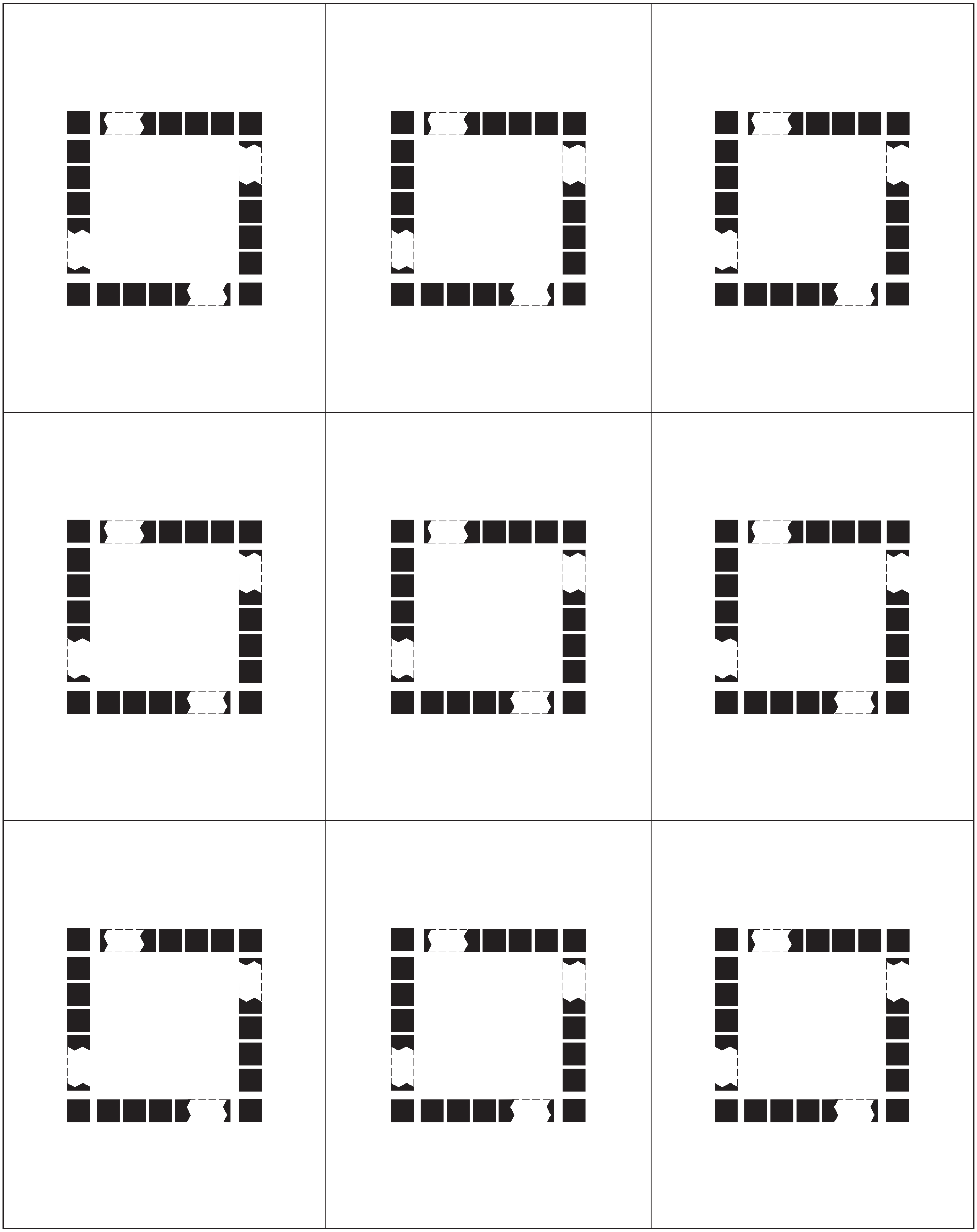
…

Arrangement ___ contains ___ + 2 tile in the bottom row.

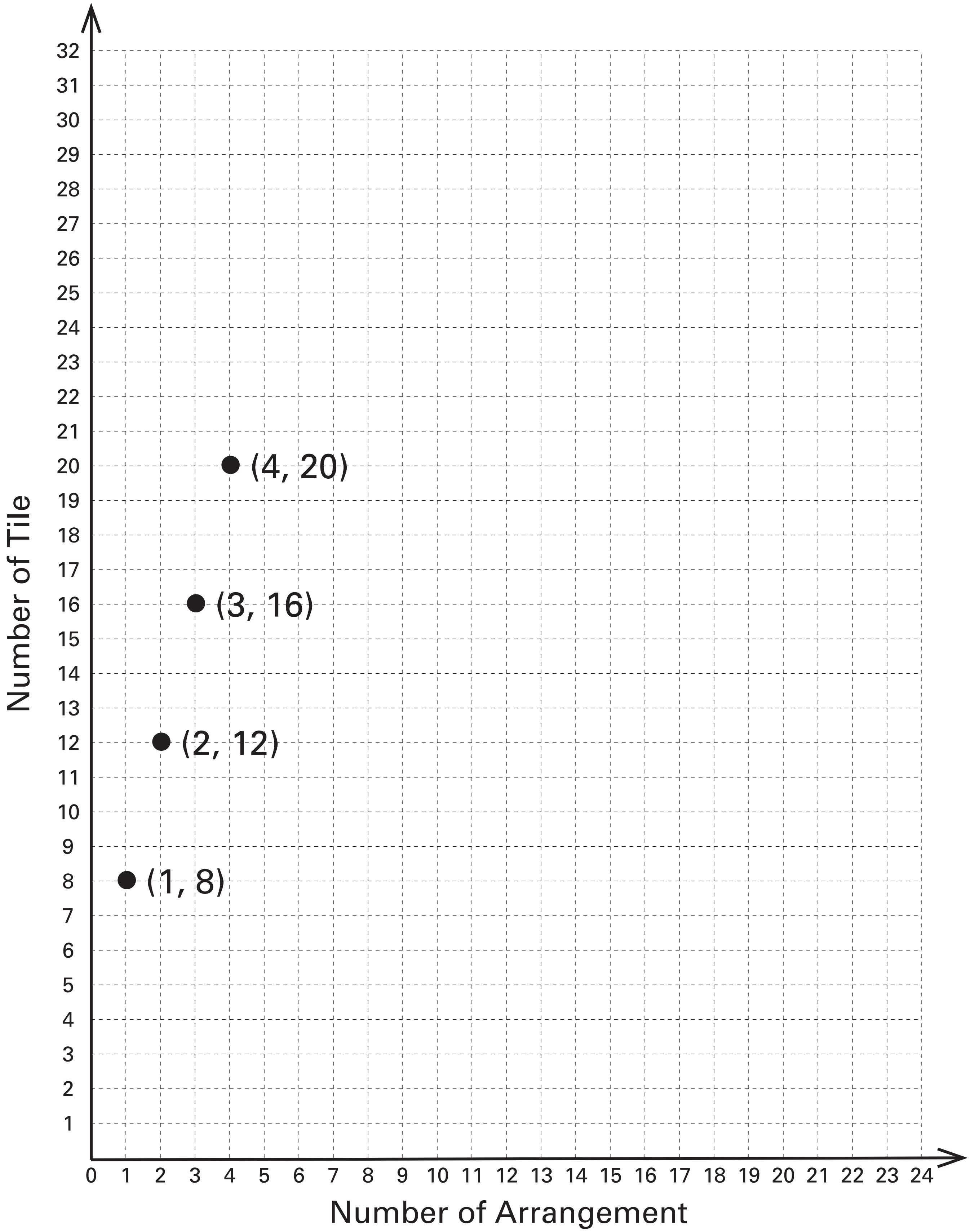
…

Arrangement *n* contains *n* + 2 tile in the bottom row.

START-UP BLACKLINE MASTER 1.3



START-UP BLACKLINE MASTER 1.4



FOCUS

Overview

Tile patterns are used to generate equivalent expressions, formulate equations, solve equations, and introduce coordinate graphs.

Materials

- Red and black counting pieces, 25 per student.
- $\frac{1}{4}''$ grid paper (see Appendix), 2 sheets per student and 1 transparency.
- Focus Master 1.1-1.2, 1 copy of each per student and 1 transparency.
- Focus Master 1.3, 1 copy per student.
- Black counting pieces for the overhead.

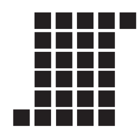
ACTIONS

1 Distribute counting pieces to each student or group of students. Display the following sequence of 4 tile arrangements on the overhead. Have the students form this sequence of arrangements. Then have them form the next arrangement in the sequence.

2 Distribute a copy of Focus Master 1.1 to the students. Ask the students to consider the sequence of arrangements in which the 5th arrangement is the one illustrated in Comment 1. Ask them to determine a variety of ways to view the 20th arrangement and to determine the number of tile required to build it, and to record their methods on Focus Master 1.1 (see following page). Place a transparency of Focus Master 1.1 on the overhead and ask for volunteers to describe their methods.

COMMENTS

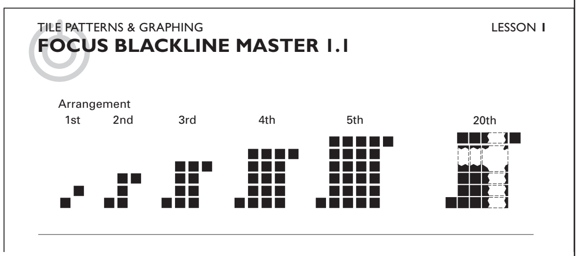
1 Here is the most frequently suggested 5th arrangement. Acknowledge other ideas suggested by students.



2 The 20th arrangement contains 401 tile. On the following page are 4 different methods of viewing the arrangements. Notice that in Method D some of the tile in the arrangements have been relocated. The students may devise other methods of viewing the arrangements.

continued next page

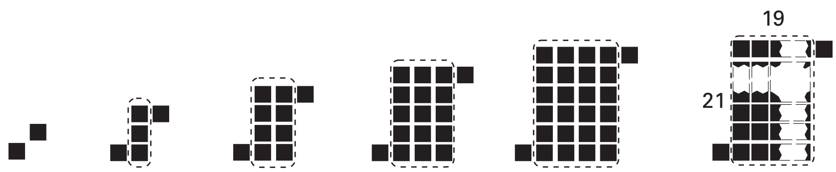
ACTIONS



COMMENTS

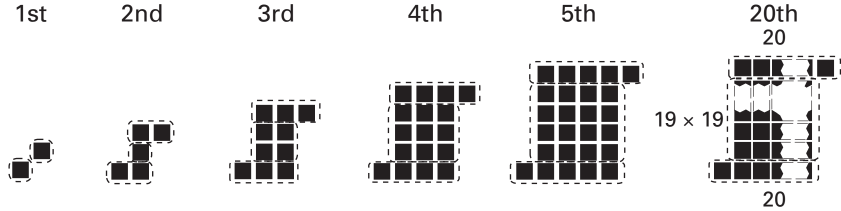
2 continued

Method A: A 19 × 21 rectangle with a single tile attached to each of the longer sides.



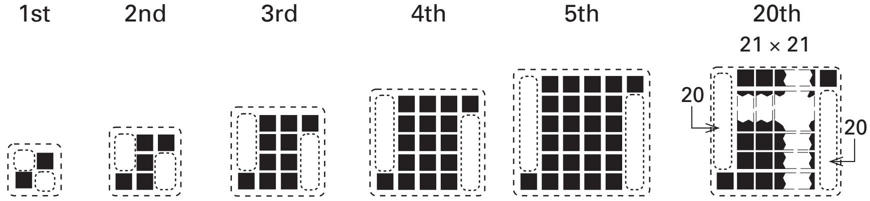
1st	2nd	3rd	4th	5th	20th
2	$(1 \times 3) + 2$	$(2 \times 4) + 2$	$(3 \times 5) + 2$	$(4 \times 6) + 2$	$(19 \times 21) + 2$

Method B: A 19 × 19 square with a row of 20 tile on the top and another row of 20 tile on the bottom.



1st	2nd	3rd	4th	5th	20th
$2(1)$	$1^2 + 2(2)$	$2^2 + 2(3)$	$3^2 + 2(4)$	$4^2 + 2(5)$	$19^2 + 2(20)$

Method C: A 21 × 21 square with 20 tile removed from the first and last columns.



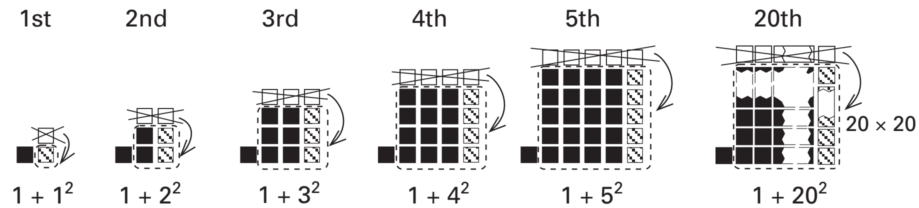
1st	2nd	3rd	4th	5th	20th
$2^2 - 2(1)$	$3^2 - 2(2)$	$4^2 - 2(3)$	$5^2 - 2(4)$	$6^2 - 2(5)$	$21^2 - 2(20)$

FOCUS

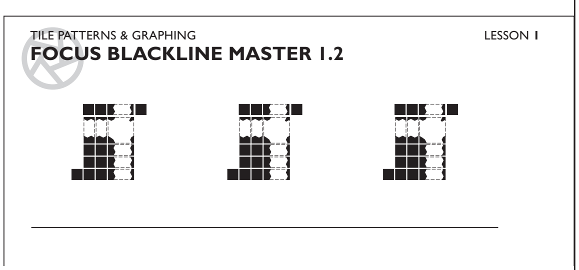
ACTIONS

COMMENTS

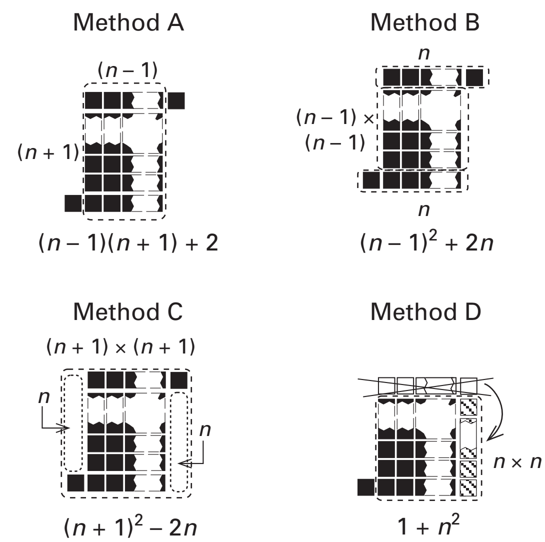
Method D: A 20 × 20 square with a single tile attached to the lower left corner.



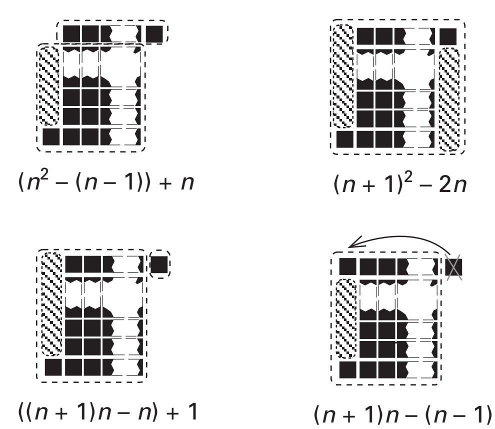
3 Distribute a copy of Focus Master 1.2 to each student. Ask the students to devise methods of viewing the *n*th arrangement of the sequence. For each method, ask the students to illustrate that method of viewing the arrangement and write a formula for the number of tile in the *n*th arrangement which reflects that method of viewing the arrangement. Place a transparency of Focus Master 1.2 on the overhead and ask for volunteers to show their methods.


TILE PATTERNS & GRAPHING LESSON 1
FOCUS BLACKLINE MASTER 1.2

3 Shown below are the *n*th arrangements corresponding to the 4 ways of viewing the arrangements shown above.



In addition to the methods shown above. Other methods are possible. Shown below are methods which few view the *n*th arrangement as a configuration from which tile have been removed. The regions from which tile have been removed are shaded.

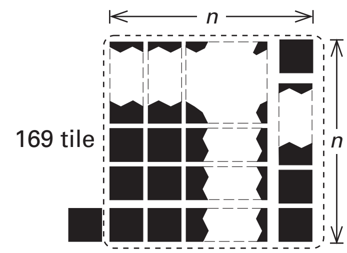


ACTIONS

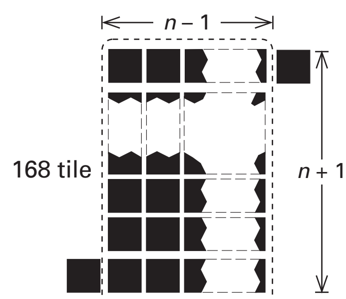
4 Ask the students to determine which arrangement contains 170 tile. Discuss the methods they use.

COMMENTS

4 If the arrangement is thought of in the manner of Method D on the previous page, one of the 170 tile would be attached to a square formed with the remaining 169. The side of this square, 13, is the number of the arrangement. A solution to the equation $n^2 + 1 = 170$ has been found.



Thinking about the arrangement in the manner of Method A on the previous page, 2 of the 170 tile are attached to a rectangle formed by the remaining 168. The dimensions of this rectangle differ by 2. Examining factors of 168, one finds the dimensions are 12 and 14. Since the number of the arrangement is 1 more than the smaller of these numbers (or 1 less than the greater), it is 13. Note that a number n has been found, namely 13, such that $(n - 1)(n + 1) + 2 = 170$.



FOCUS

ACTIONS

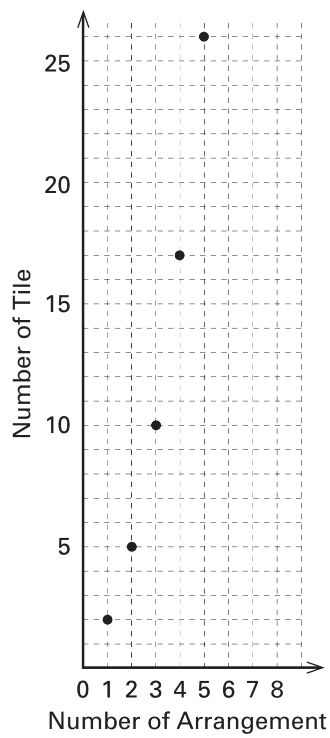
5 Distribute ¼″ grid paper to the students. Ask them to construct and label a coordinate graph which shows the number of tile in the first 5 arrangements. Ask the students for their observations.

6 Distribute a copy of Focus Master 1.3 to each student (see following page). Ask the students to complete parts a) and b). Discuss the students' responses, in particular, ask for volunteers to show the sketches they made in part b). Then ask the students to complete the remaining parts. Discuss their results and the methods used to arrive at them.

COMMENTS

5 A coordinate graph of the first 5 arrangements is shown below.

Here are some possible observations.



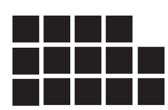
The points do not lie on a line.

The vertical distant between points increases as the number of arrangement increases.

The vertical distance between points goes up by 2 as we move from point to point; at first, it's 3, then it's 5, then 7, and so forth.

6 Parts of this activity could be assigned as homework.

a) Here is the most frequently suggested 4th arrangement:



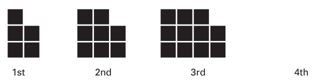
continued next page

ACTIONS

TILE PATTERNS & GRAPHING — LESSON 1

FOCUS BLACKLINE MASTER 1.3

a) Draw the next arrangement in the following sequence:



b) How many tile does the 40th arrangement contain? Draw a rough sketch or diagram that shows how you arrived at your answer.

c) Find at least 2 different expressions for the number of tile in the *n*th arrangement. For each expression, draw a rough sketch or diagram that shows how you arrived at that expression.

d) Which arrangement contains exactly 500 tile? Draw a rough sketch or diagram of this arrangement.

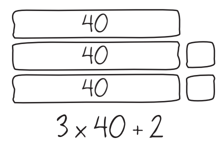
e) On a sheet of 1/4″ grid paper, construct and label a coordinate graph showing the number of tile in each of the first 8 arrangements.

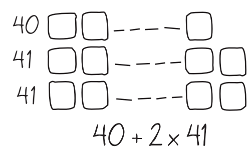
f) (Challenge) Two arrangements together contain 160 tile. One of the arrangements contains 30 more tile than the other. Draw a rough sketch or diagram of these 2 arrangements. Which arrangements are these?

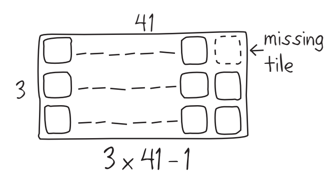
COMMENTS

6 continued

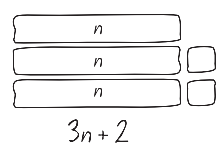
b) There are 122 tile in the 40th arrangement. The students may have difficulty drawing simple sketches which depict their method of viewing their arrangement. Initially, they may include more detail than necessary. Here are some possibilities:

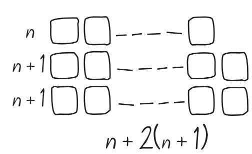


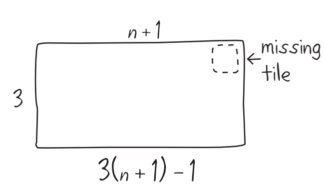




c) Here are some sketches of the *n*th arrangement:





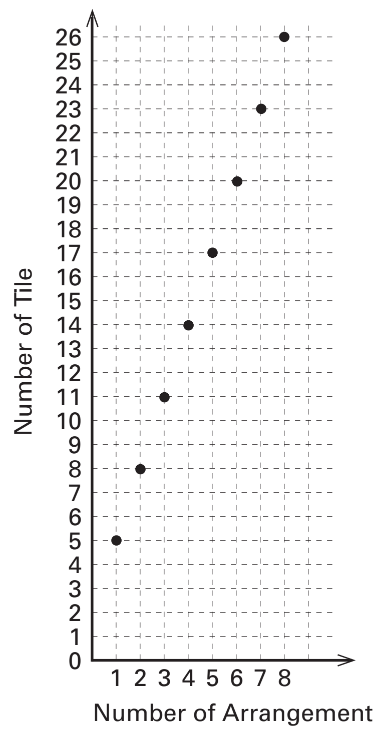


ACTIONS

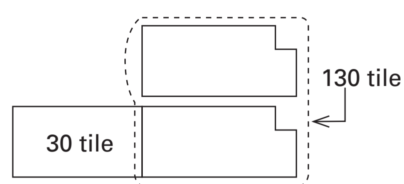
COMMENTS

d) If an arrangement contains 500 tile, the top row contains 498 ÷ 3, or 166, tile. The number of tile in the top row is the same as the number of the arrangement.

e) Below is a graph showing the number of tile in each of the first 8 arrangements.



f) As shown in the figure below, the smaller arrangement contains 130 ÷ 2, or 65, tile. An arrangement with 65 tile has 21 tile in the top row and hence is the 21st arrangement. The larger arrangement has 30 ÷ 3 = 10 more tile in the top row and, hence, is the 31st arrangement.

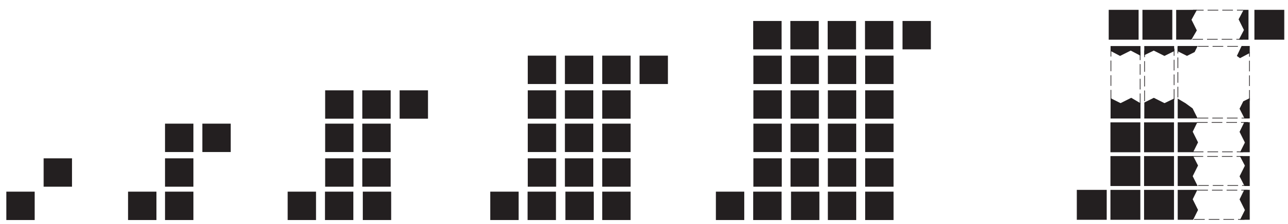


TEACHER NOTES

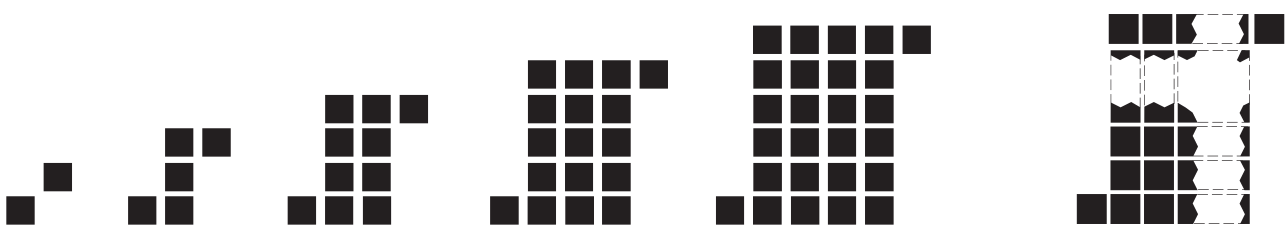
FOCUS BLACKLINE MASTER 1.1

Arrangement
1st 2nd 3rd 4th 5th 20th

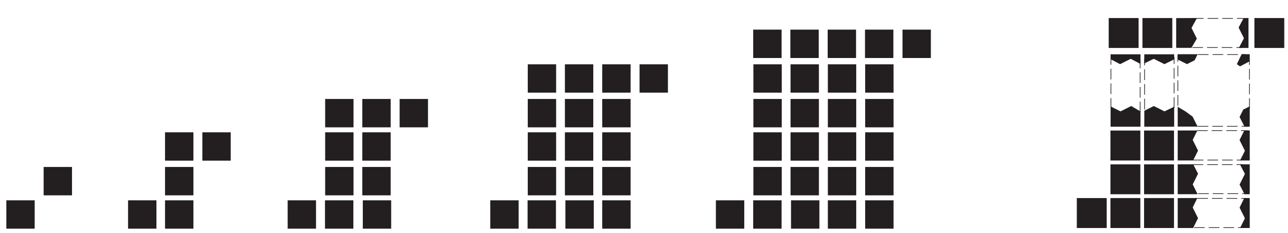
Arrangement
1st 2nd 3rd 4th 5th 20th



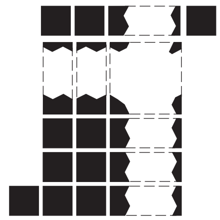
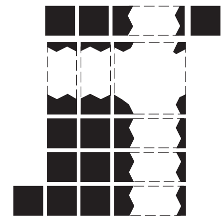
Arrangement
1st 2nd 3rd 4th 5th 20th

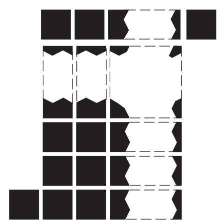


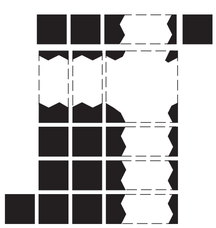
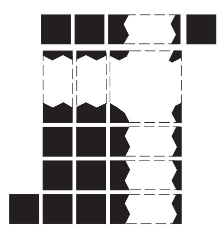
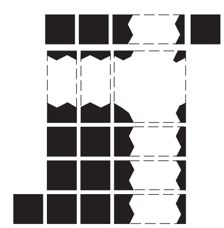
Arrangement
1st 2nd 3rd 4th 5th 20th

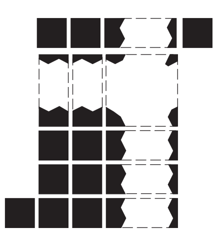
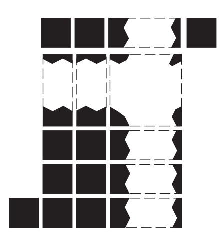
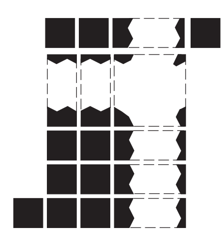


FOCUS BLACKLINE MASTER 1.2

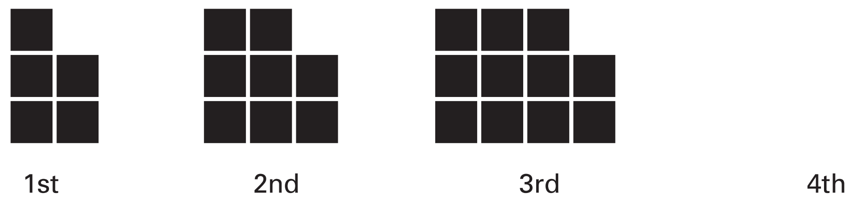



FOCUS BLACKLINE MASTER 1.3

a) Draw the next arrangement in the following sequence:



b) How many tile does the 40th arrangement contain? Draw a rough sketch or diagram that shows how you arrived at your answer.

c) Find at least 2 different expressions for the number of tile in the *n*th arrangement. For each expression, draw a rough sketch or diagram that shows how you arrived at that expression.

d) Which arrangement contains exactly 500 tile? Draw a rough sketch or diagram of this arrangement.

e) On a sheet of ¼″ grid paper, construct and label a coordinate graph showing the number of tile in each of the first 8 arrangements.

f) (Challenge) Two arrangements together contain 160 tile. One of the arrangements contains 30 more tile than the other. Draw a rough sketch or diagram of these 2 arrangements. Which arrangements are these?

FOLLOW-UP BLACKLINE MASTER 1

1 Shown here are the first 3 arrangements in a sequence of tile arrangements.

a) Describe, in words only, the 50th arrangement so anyone who reads your description could build it.

b) Determine the number of tile in the 50th arrangement. Draw a rough sketch or diagram that shows how you determined the number.

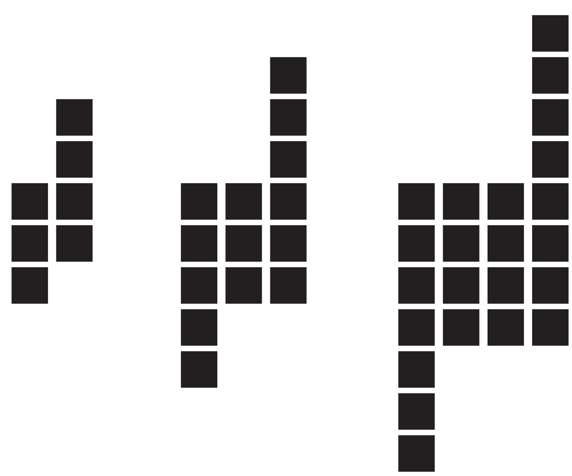
c) Find at least 2 different expressions for the number of tile in the *n*th arrangement. Draw rough sketches or diagrams to show how you obtained these expressions.

d) On a sheet of ¼″ grid paper, draw a graph showing the number of tile in the first several arrangements of the above sequence.

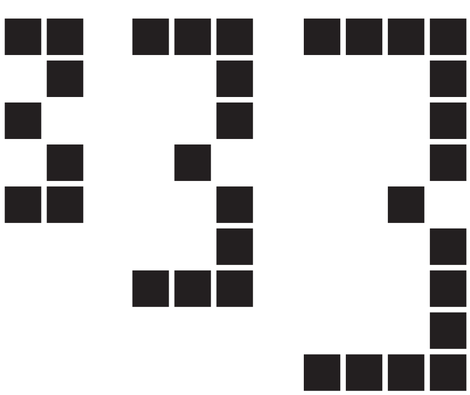
2 Repeat parts b), c), and d) above for each of the following sequences of tile arrangement.

I

II



III



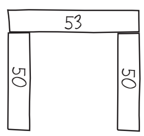
ANSWERS TO FOLLOW-UP 1

1 a) Some possible descriptions of the 50th arrangement:

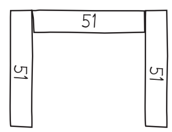
"A row of 53 tile with columns of 50 tile added under the first and last tiles in the row."

"2 columns of 51 tile with a row of 51 tile added between the top tiles of the 2 columns."

b) The 50th arrangement contains 153 tile.

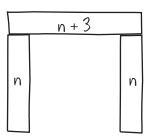


$53 + 2(50)$

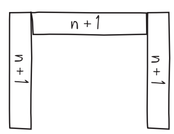


3×51

c) Possible ways of viewing *n*th arrangement:

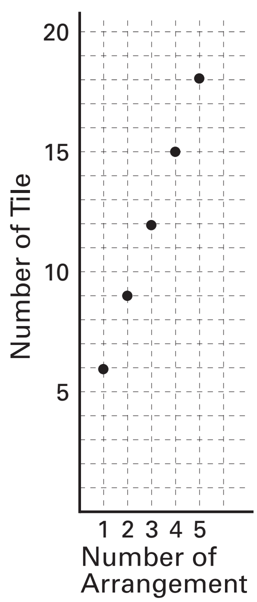


$(n + 3) + 2n$



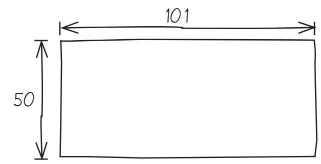
$3(n + 1)$

d)

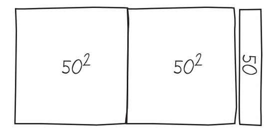


2 1 The 50th arrangement contains 5050 tile.

b) Possible ways of viewing 50th arrangement:

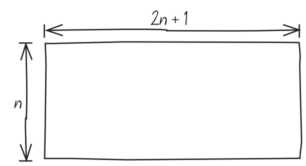


50×101

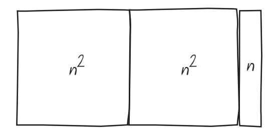


$2(50^2) + 50$

c) Possible ways of viewing *n*th arrangement:

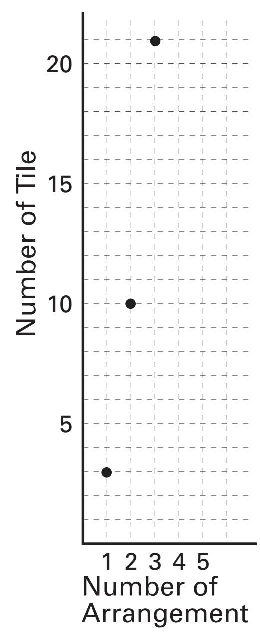


$n(2n + 1)$



$2n^2 + n$

d)



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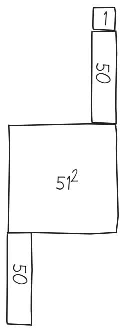
ANSWERS TO FOLLOW-UP 1 (CONT.)

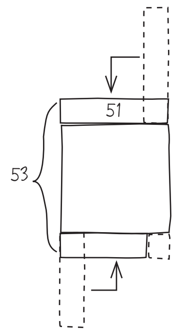
2 continued

II The 50th arrangement contains 203 tile.

b) Possible ways of viewing 50th arrangement:

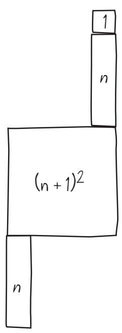


$51^2 + 2(50) + 1$

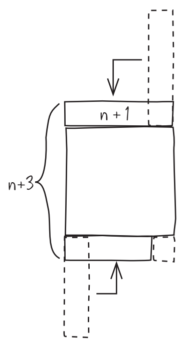


$(53)(51) - 1$

c) Possible ways of viewing *n*th arrangement:

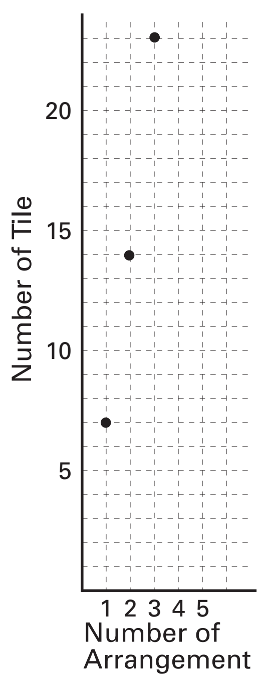


$(n + 1)^2 + 2n + 1$



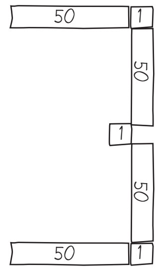
$(n + 3)(n + 1) - 1$

d)

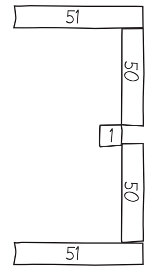


III The 50th arrangement contains 2702 tile.

b) Possible ways of viewing 50th arrangement:

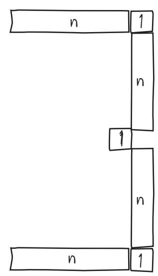


$4 \times 50 + 3$

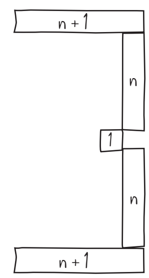


$2(50 + 51) + 1$

c) Possible ways of viewing *n*th arrangement:



$4n + 3$



$2(n + 1 + n) + 1$

d)